Nuclear Structure from Gamma-Ray Spectroscopy

2019 Postgraduate Lectures

Lecture 1: What we can learn about nuclear behaviour from gamma-ray spectroscopy

Nuclear Physics

- The nucleus is one of nature's most interesting quantal few-body systems

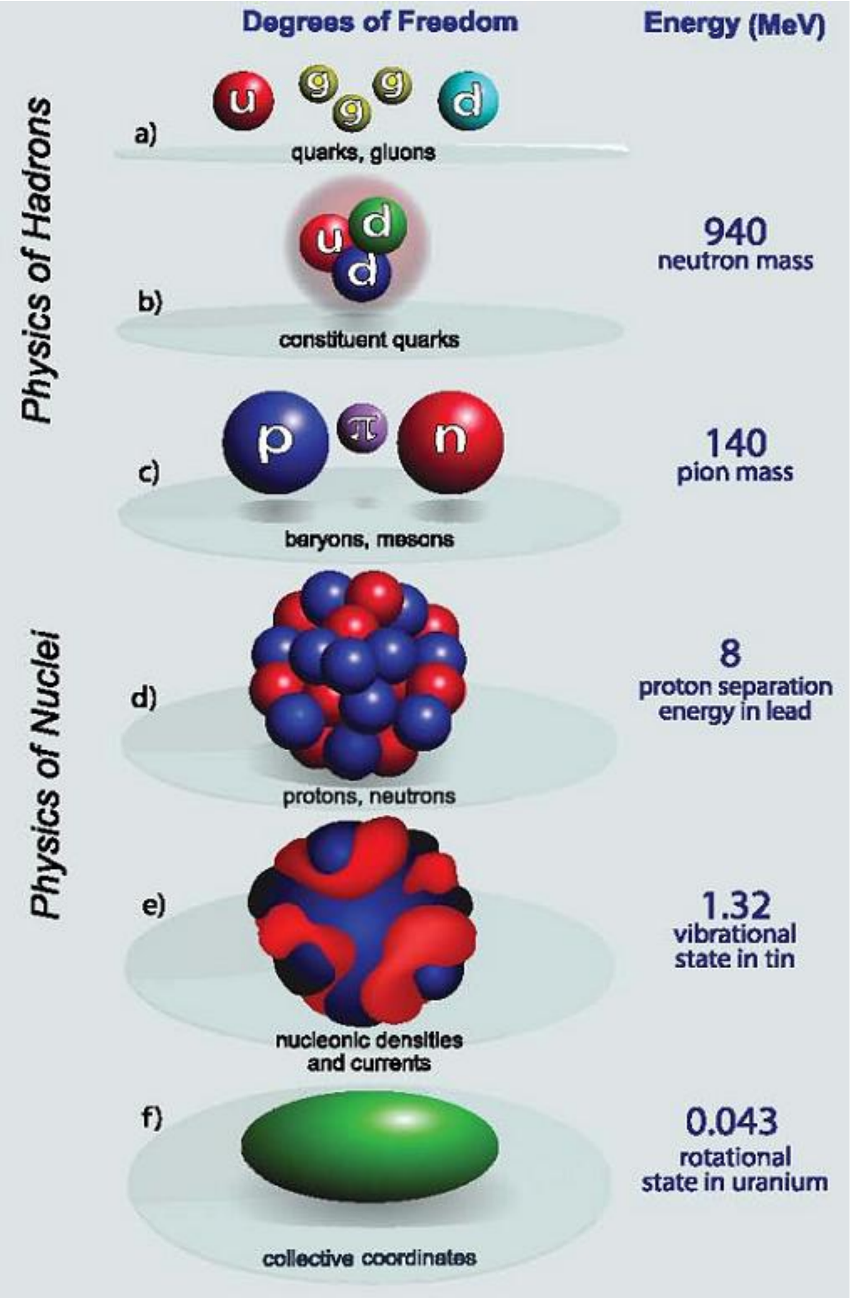
- It brings together many types of behaviour, almost all of which are found in other systems but which in nuclei interact with one another

- The major elementary excitations in nuclei can be associated with single-particle or collective modes

- While these modes can exist in isolation, it is the interaction between them that gives nuclear spectroscopy its rich diversity

The Nucleus is Unique!

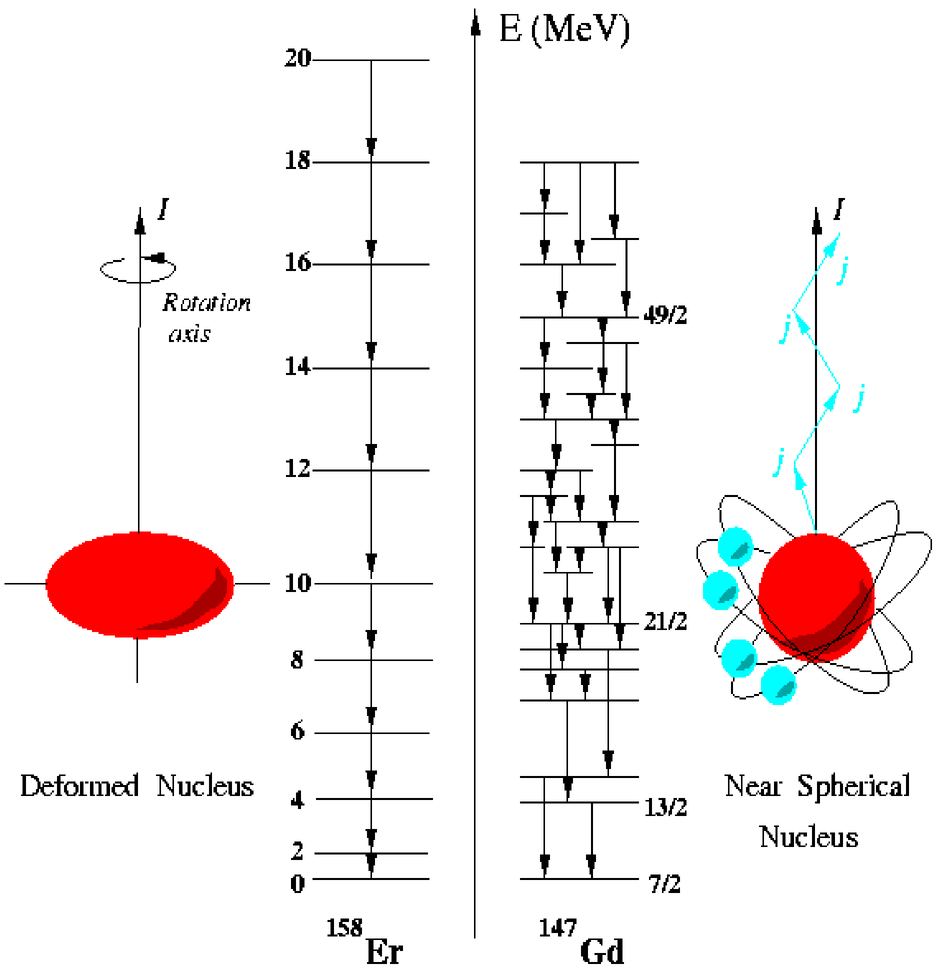
- The nucleus is a unique ensemble of strongly interacting fermions (nucleons)
- Its large, yet finite, number of constituents controls the physics of this mesoscopic system
- Both single-particle (out-of-phase) and collective (in-phase) effects occur
- There is an analogy to a herd of wild animals. Individual animals may break out of the herd but are rapidly drawn back to the safety of the collective



Building Blocks and Energy Scales

- Depending on energy and length scales, different constituents may be considered as the building blocks of the atomic nucleus

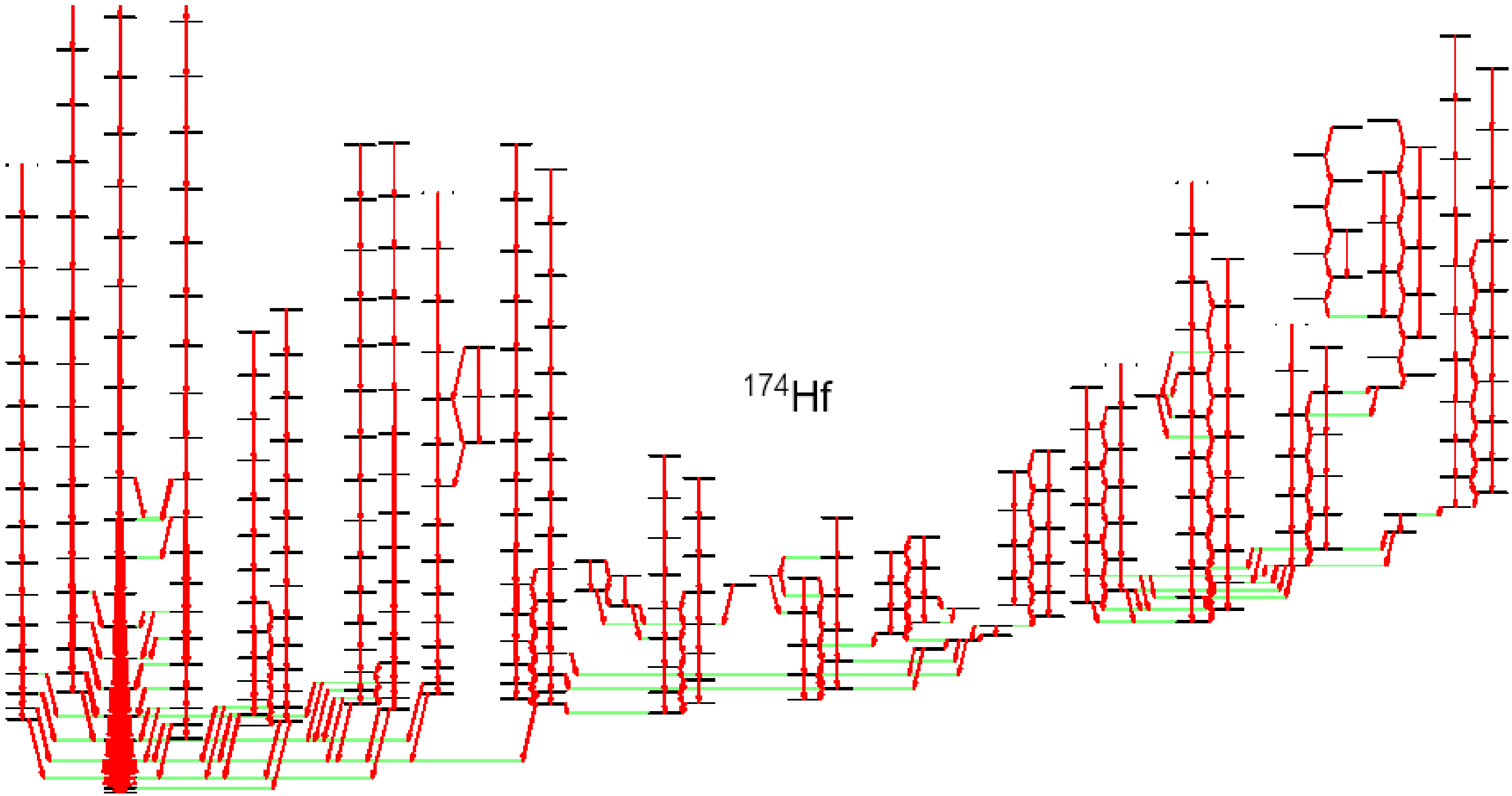
Generation of Angular Momentum



- There are two basic ways of generating high-spin states in a nucleus

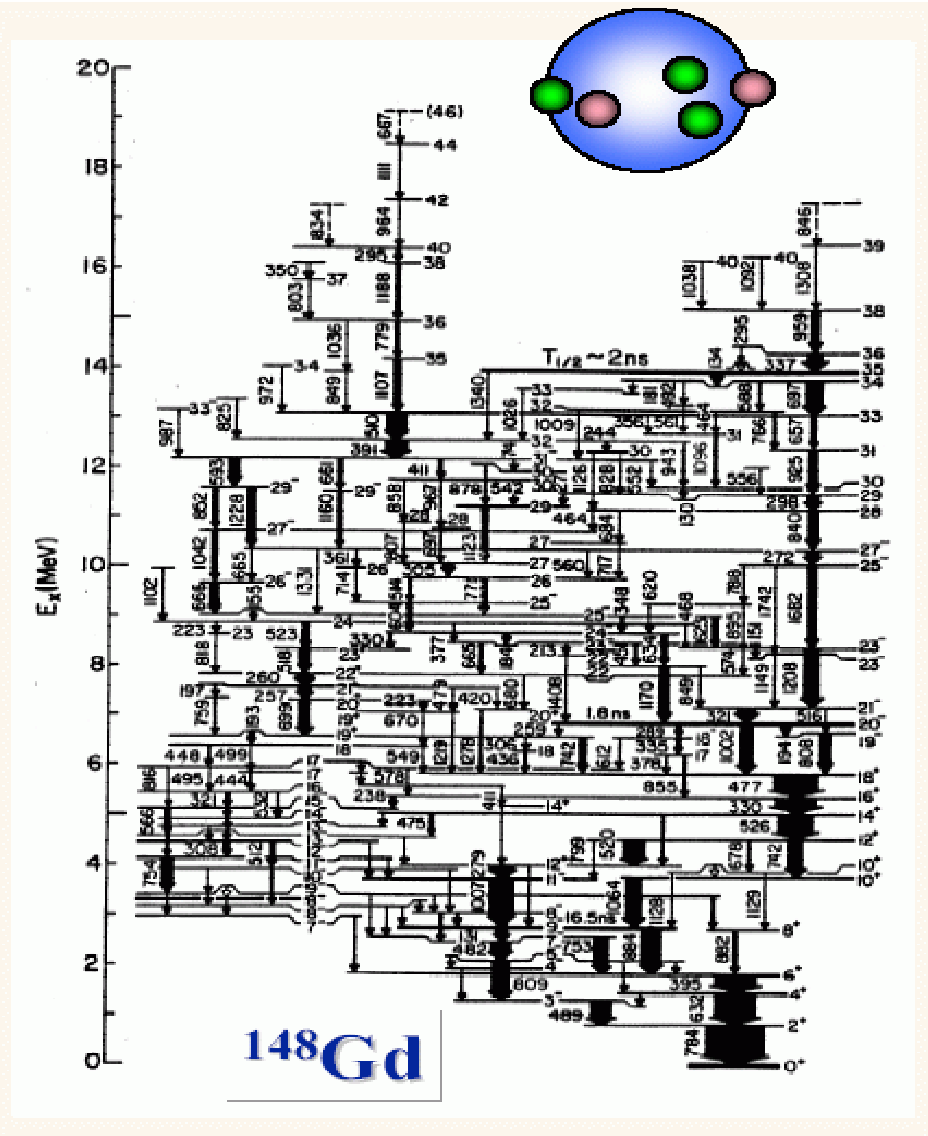
1. Collective (in-phase) motions of the nucleons: vibrations, rotations etc
2. Single-particle effects: pair breaking, particle-hole excitations. The individual spins of a few nucleons j_i generate the total nuclear spin

Collective Level Scheme



- This nucleus has 347 known levels and 516 gamma rays !

Noncollective Level Scheme



- ^{148}Gd is an example of a nucleus showing single-particle behaviour
- Complicated set of energy levels
- No regular features e.g. band structures
- Some states are isomeric

Do Nuclei Really Rotate?

- Should we talk about collective motion in nuclei?
- We need to identify fast and slow degrees of freedom
- For example, in molecules electronic motion is the fastest, vibrations are 10^2 times slower and rotations 10^6 times slower. These motions have very different time scales so the wavefunction can be separated into a product of the terms
- For nuclei the differences are much smaller. Collective and single-particle modes can perhaps be separated, but they will interact strongly !

Nuclear Deformation

- Shape parameterisation
- Quadrupole deformation, β and γ
- Triaxiality

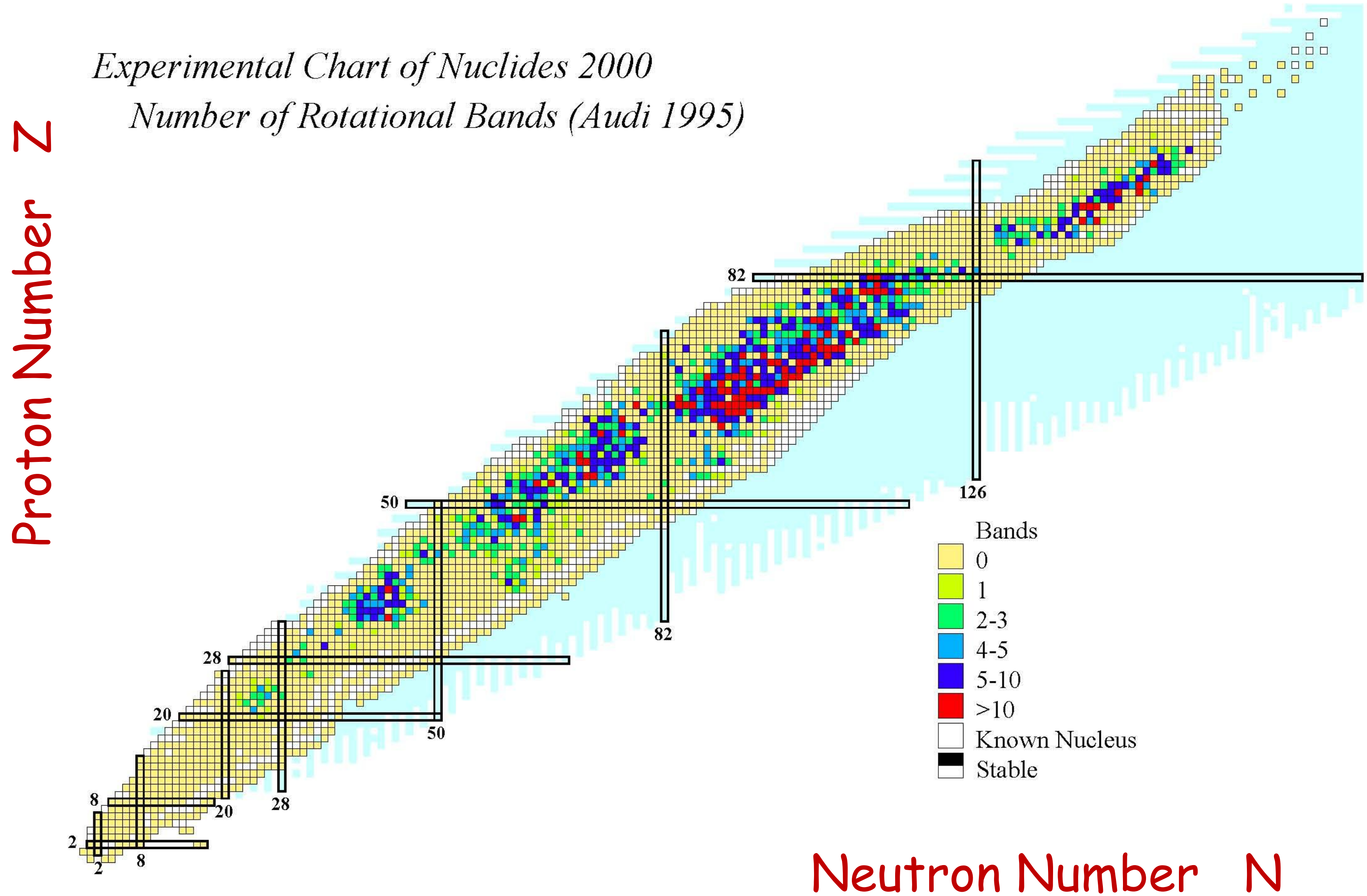
Evidence for Deformation

1. Large electric quadrupole moments Q_0

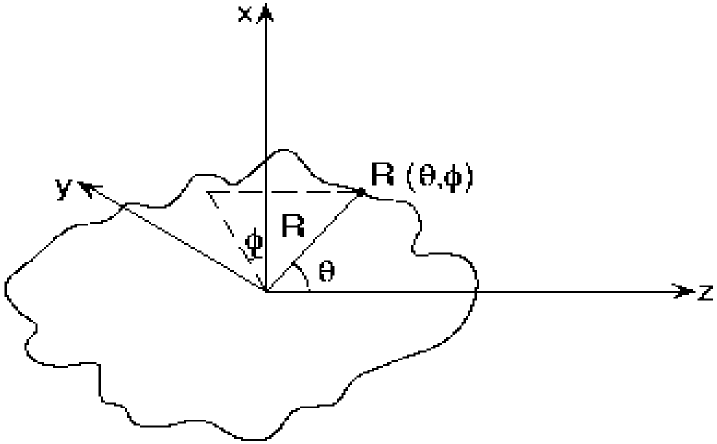
2. Low-lying rotational bands ($E \propto I[I+1]$)

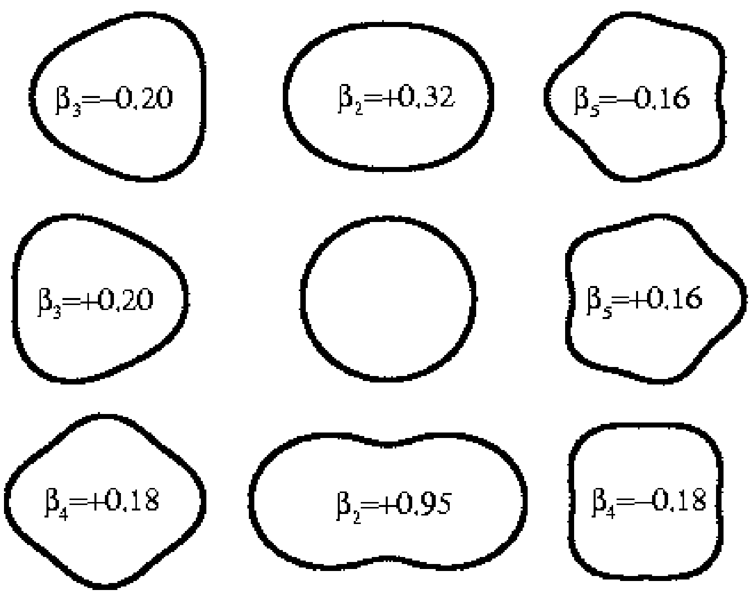
The origin of deformation lies in the long range component of the nucleon-nucleon residual interaction: a quadrupole-quadrupole interaction gives increased binding energy for nuclei which lie between closed shells if the nucleus is deformed. In contrast, the short range (pairing) component favours sphericity

Deformation: Rotational Bands



Simple Nuclear Shapes





- In the description of a 'drop' of nuclear matter with a sharp surface, the equipotential surface $R(\theta,\varphi)$ can be expressed as a sum over spherical harmonics $Y_{\lambda\mu}(\theta,\varphi)$:

$$R(\theta,\varphi) = R_0[1 + \sum_\lambda \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\theta,\varphi)]$$

- Here R_0 is the radius of a sphere and the $\alpha_{\lambda\mu}$ coefficients represent distortions from the equilibrium spherical shape

Volume Conservation

- By integrating over the shape of the nucleus, the volume for small deformation is:

$$V \approx (4\pi/3)\,[1 + 3\alpha_{00}/\sqrt{(4\pi)}]\;R_0^3$$

- To account for the incompressibility of nuclear matter we demand volume conservation under distortions and hence set $\alpha_{00} = 0$

- A factor $C(\alpha_{\lambda\mu})$ may be introduced to satisfy the conservation of volume more precisely:

$$R(\theta,\varphi) = C(\alpha_{\lambda\mu})\,R_0[1 + \Sigma_\lambda\Sigma_\mu\alpha_{\lambda\mu}\,Y_{\lambda\mu}(\theta,\varphi)]$$

Most Important Multipoles

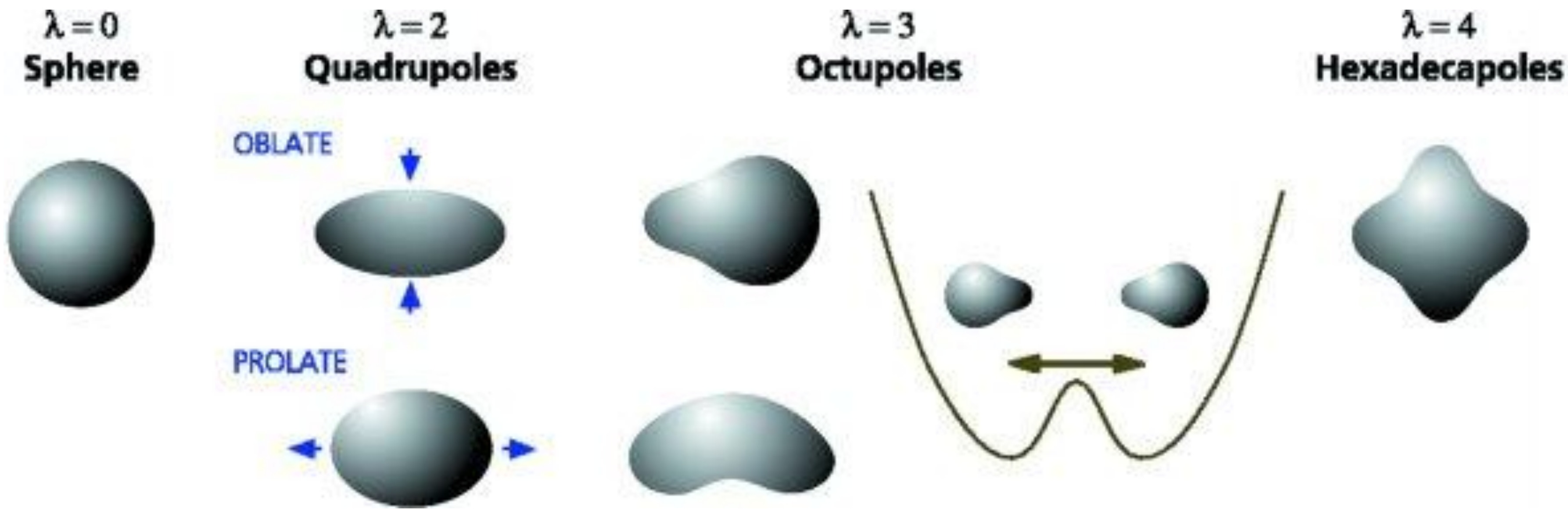
- The $\lambda = 1$ term describes the displacement of the centre of mass and therefore cannot give rise to intrinsic excitation of the nucleus – ignore !

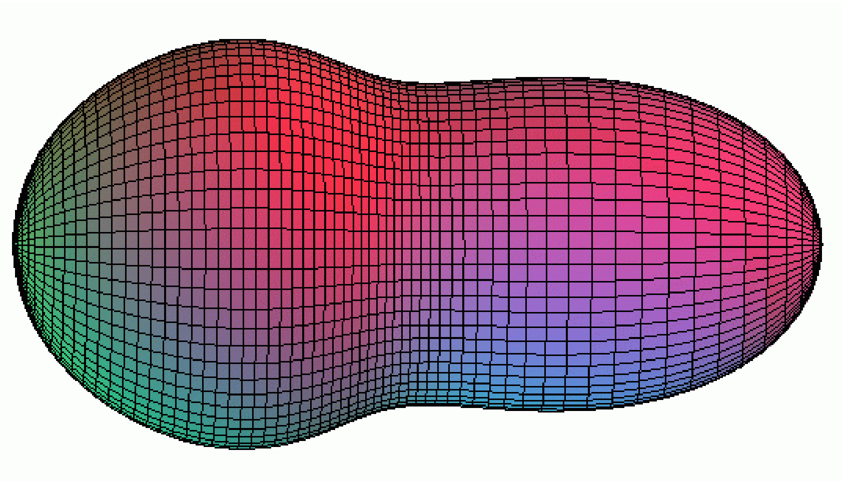
- The $\lambda = 2$ term is the most important term and describes quadrupole deformation

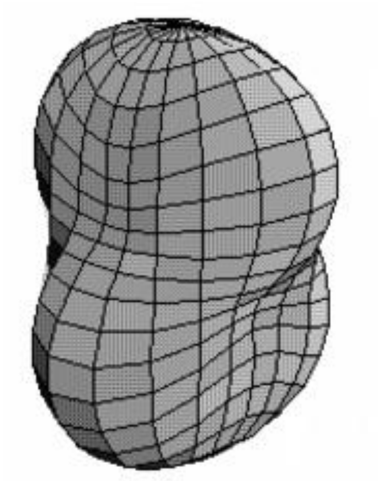
- The $\lambda = 3$ term describes octupole shapes which can look like pears ($\mu = 0$), bananas ($\mu = 1$) and peanuts ($\mu = 2,3$)

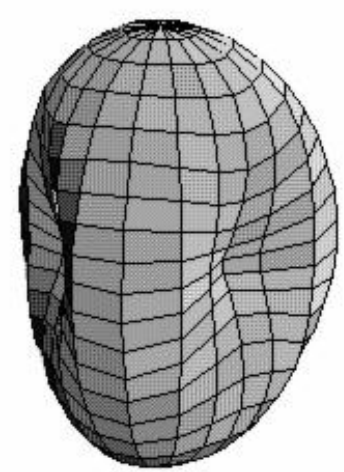
- The $\lambda = 4$ term describes hexadecapole shapes

Variety of Shapes

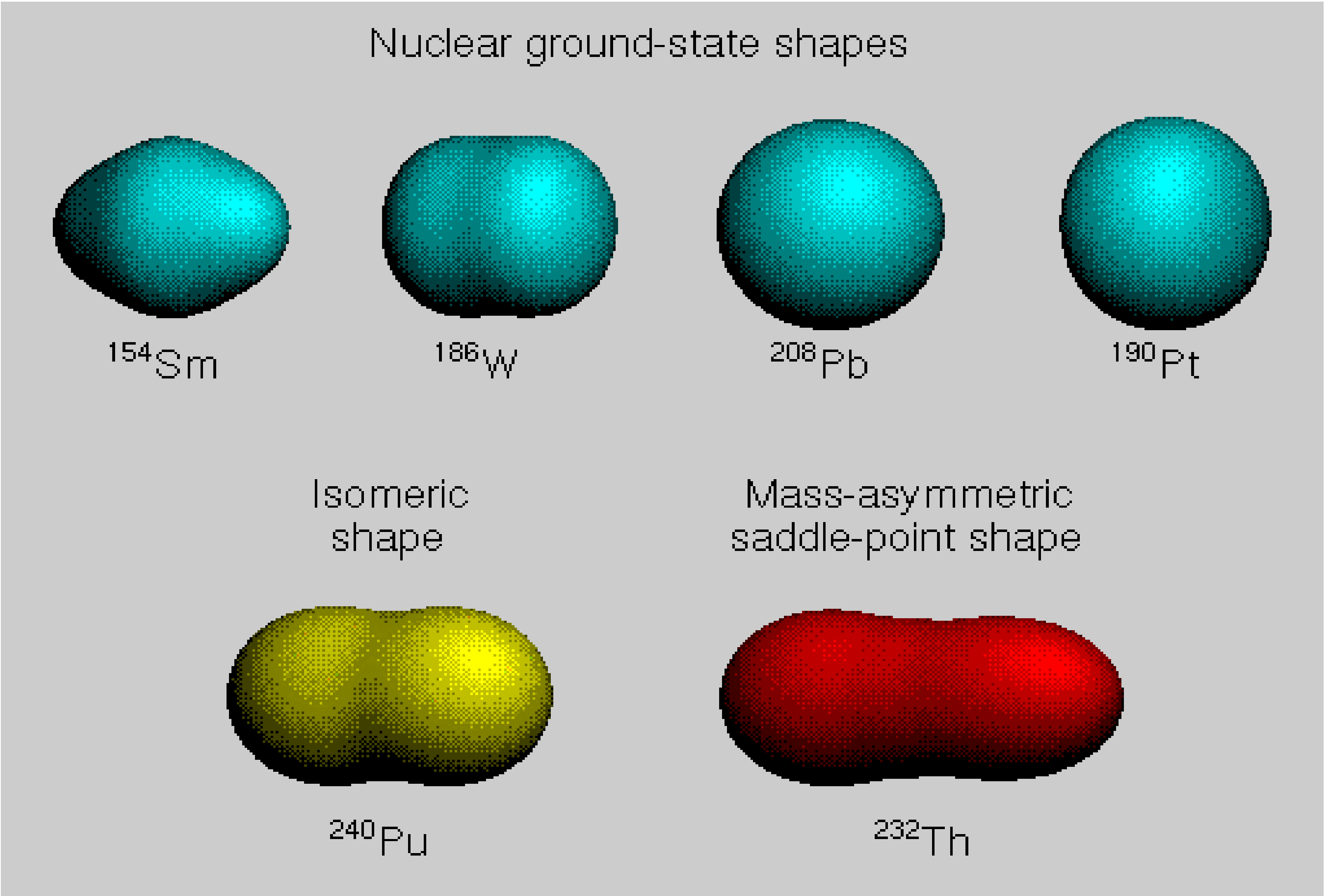








Theoretical Deformations



Tetrahedral Nuclei?

- The $\lambda = 3$ multipole describes octupole shapes

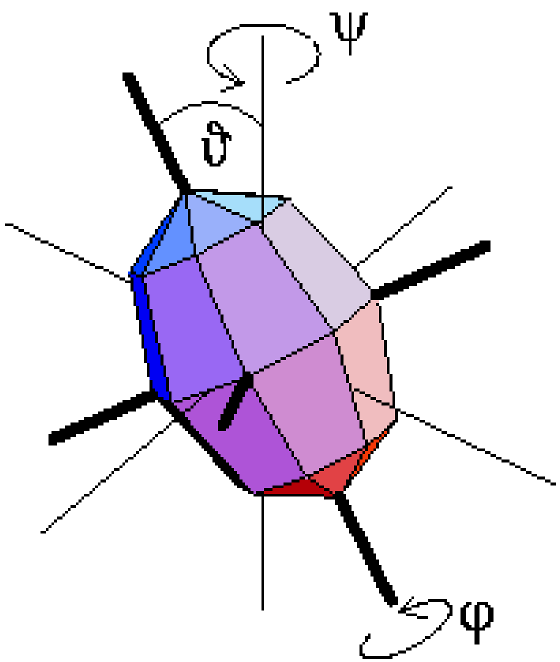
- Pear shapes (with axial symmetry) occur when

$$\alpha_{30} = \alpha_{20} \neq 0$$

- Nuclei with tetrahedral (and octahedral) symmetry have been to predicted to occur when

$$\alpha_{32} \neq 0 \quad \text{with } \alpha_{30} = \alpha_{20} \sim 0$$

Quadrupole Deformation



The Euler angles relate the intrinsic (nucleus) and lab frame axes

- The description of the nuclear shape simplifies if we make the principal axes of our coordinate system, i.e. (x, y, z), coincide with the nuclear axes (1, 2, 3)
- Then $a_{22} = a_{2-2}$, and $a_{21} = a_{2-1} = 0$
- The two independent coefficients a_{20} and a_{22}, together with the three Euler angles, then completely define the system
- The shape then simplifies to:

$$R = C\, R_0[1 + a_{20}Y_{20} + a_{22}\,(Y_{22}+Y_{2-2})]$$

β_2 and γ Parameters

- An alternative parameterisation in the system of principal axes introduces the polar coordinates (β_2, γ) through the relations:

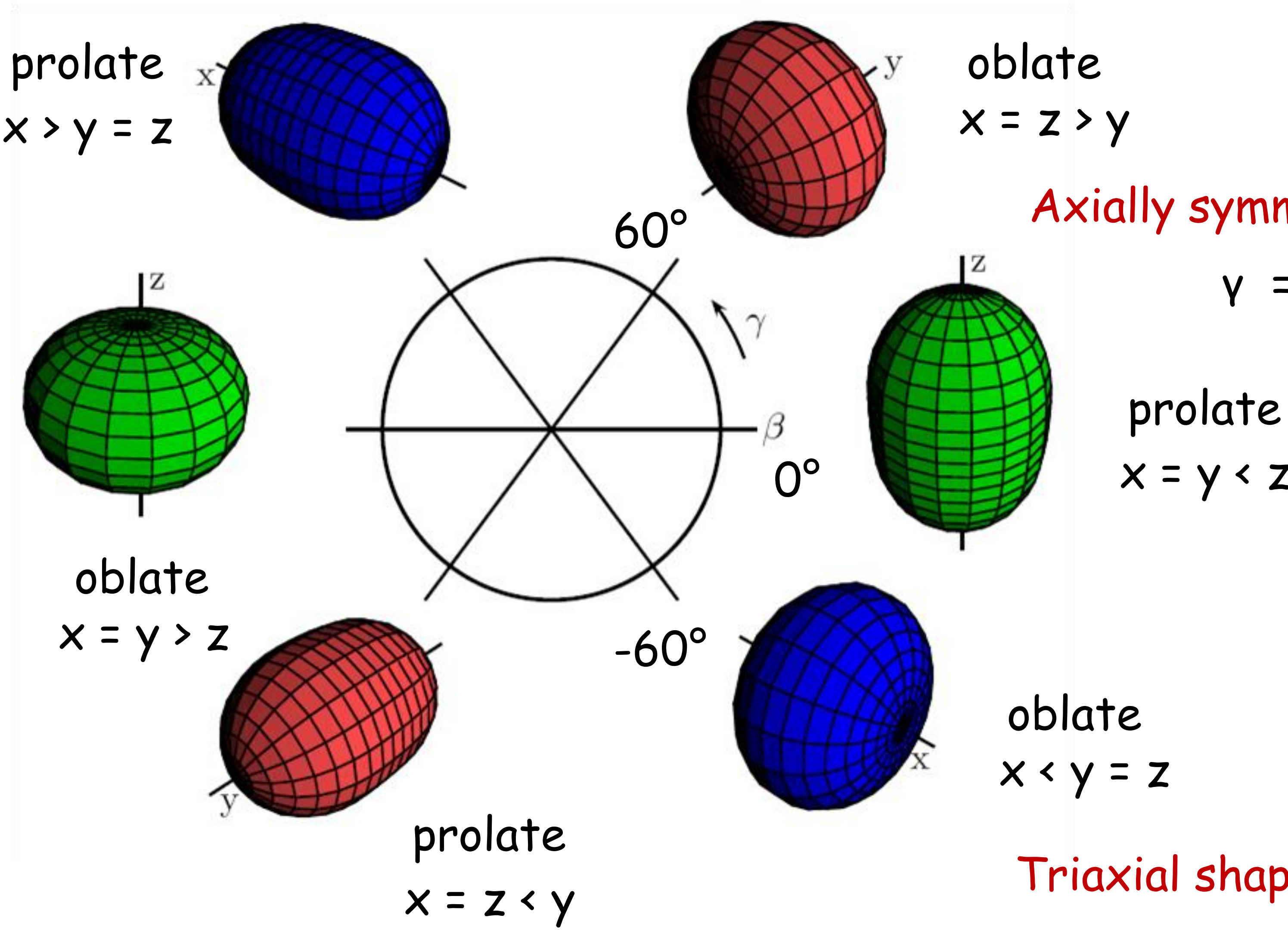
$$a_{20} = \beta_2 \cos\gamma \quad \text{and} \quad a_{22} = -1/\sqrt{2}\,\beta_2 \sin\gamma$$

- The parameter β_2 measures the total deformation:

$$\beta_2^2 = \sum_\mu |a_{2\mu}|^2$$

- The parameter γ measures the lengths along the principal axes For $\gamma = 0°$, the shape is prolate with the z-axis as the (long) symmetry axis

Quadrupole β_2 and γ Parameters



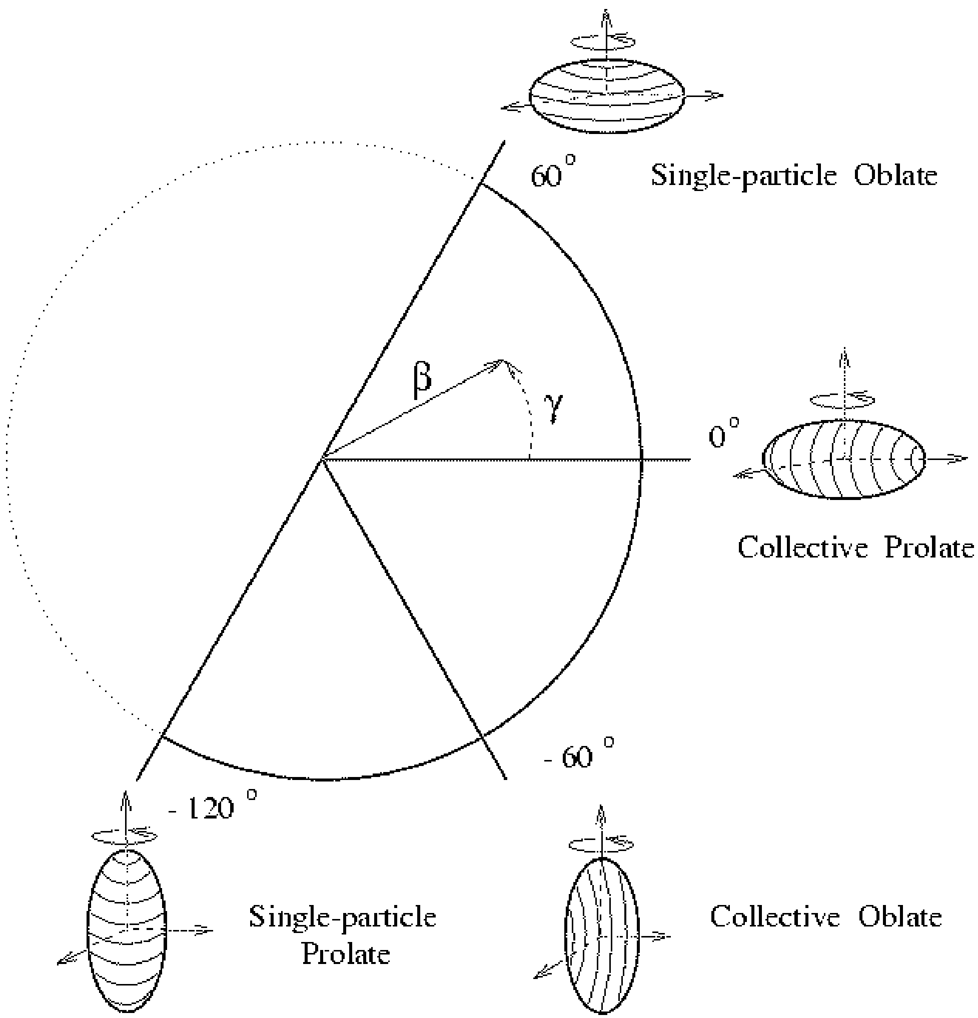
Axially symmetric shapes

$\gamma = n\,60°$

Triaxial shapes : $x \neq y \neq z$

$\gamma \neq n\,60°$

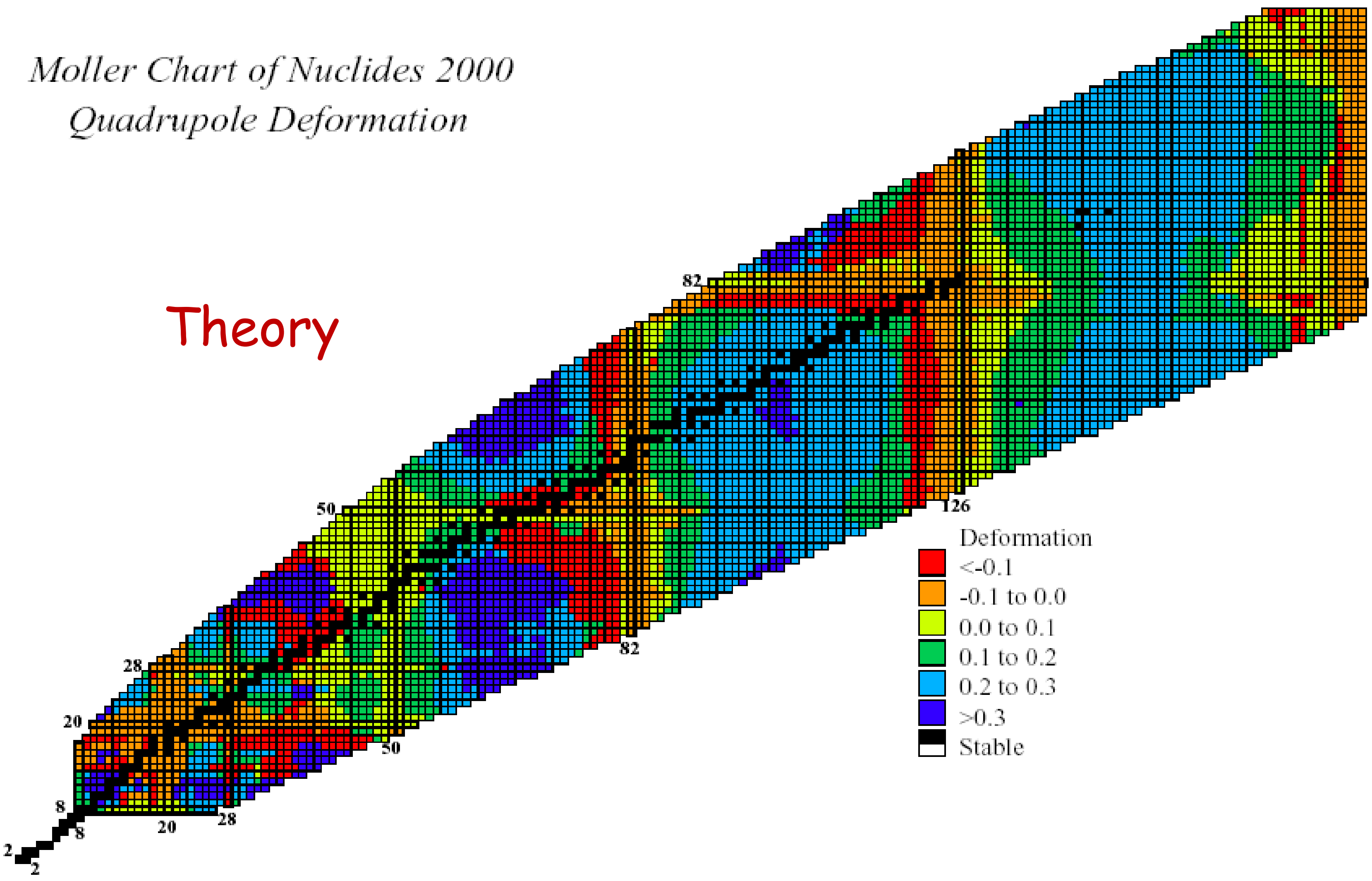
Lund Convention



- In order to specify the triaxiality of a deformed quadrupole intrinsic shape, the range of γ values,

  $0° \leq \gamma \leq 60°$ is sufficient
- However, in order to specify a cranked system, we need three times this range, corresponding to the three principal axes about which the system can be cranked

Deformation Systematics



Approximate Value of β_2

- From an empirical fit to E2 transition rates (not valid near closed shells) it is found:

$$T_\gamma(E2;\, 2^+ \rightarrow 0^+) = (4 \pm 2) \times 10^{10}\, Z^2\, E_\gamma^{\,4}\, A^{-1}$$

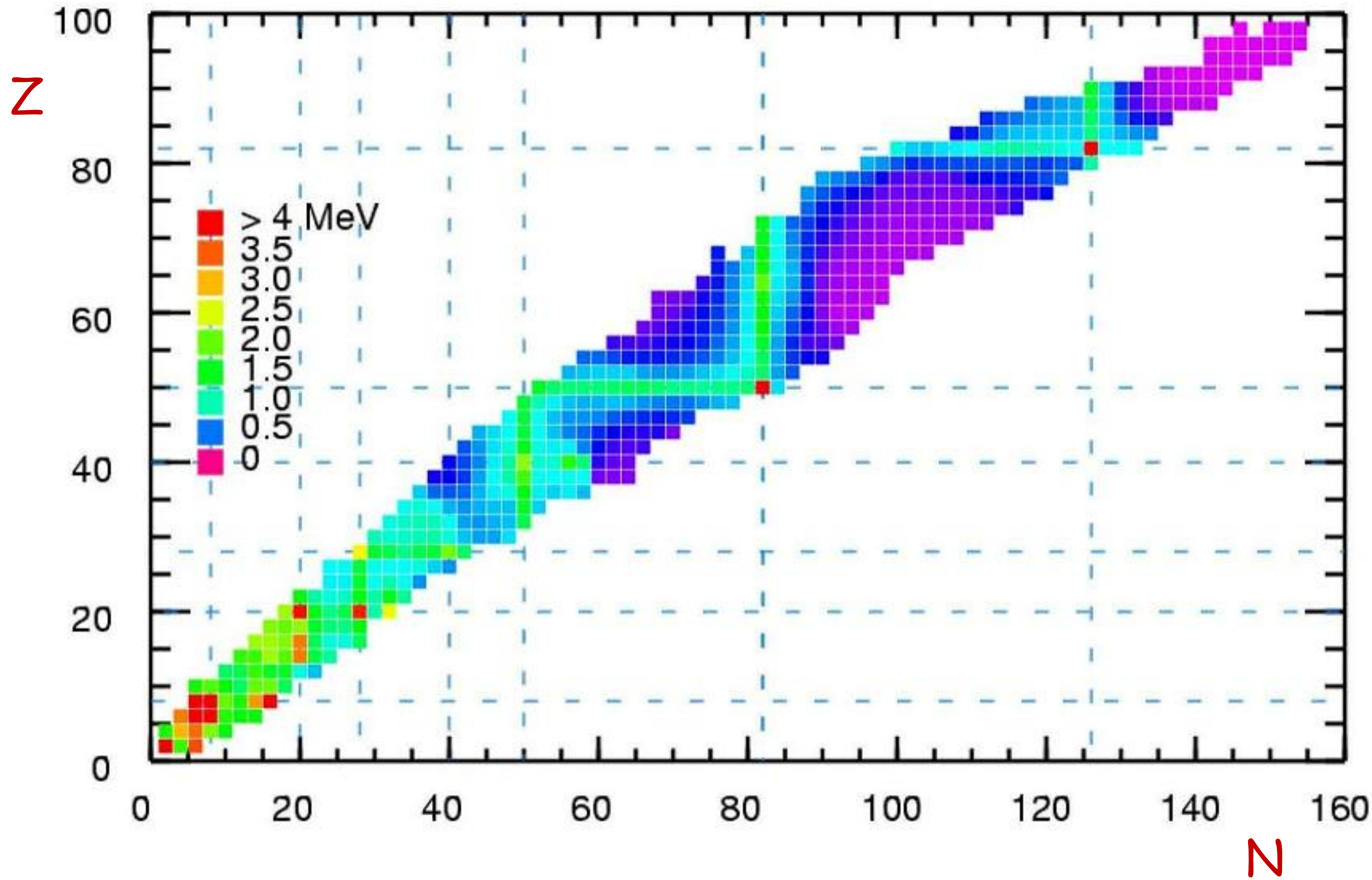
- This can be used for a 'Grodzins' estimate of the quadrupole deformation parameter:

$$\beta_2 \approx \{\, 1225 \,/\, [\, A^{7/3}\, E(2^+)\,]\, \}^{1/2}$$

with the 2^+ energy expressed in MeV

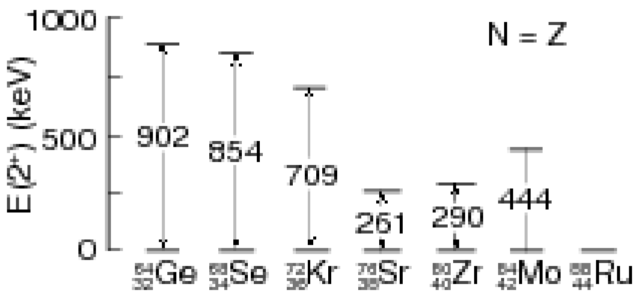
- The energy of the 2^+ state of an even-even nucleus hence gives an insight into the nuclear deformation. The lower the 2^+ energy, the larger is β_2 and also the nuclear moment of inertia

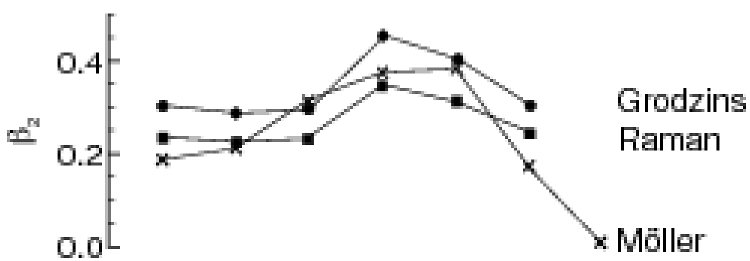
First 2^+ Energies

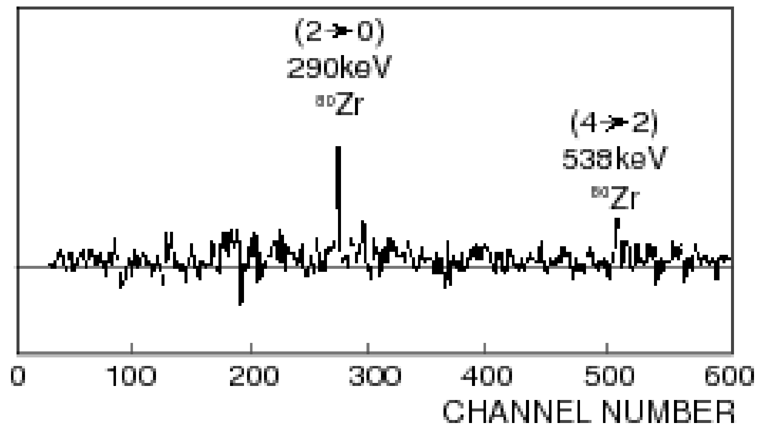


Structure Far From Stability

- Even one gamma ray can give an insight into nuclear behaviour, e.g. deformation







Triaxiality

- All three principal axes have different lengths:

  $$R_x \neq R_y \neq R_z$$

  i.e. 'short', 'long' and 'intermediate' axes

- There is no symmetry axis (however, there is reflection symmetry) so K is not a good quantum number

- The low-spin energy levels in even-even nuclei move around as a function of γ

- For $\gamma \neq 0°$ an effective quadrupole deformation parameter may be defined:

  $$\beta_{eff} = \beta \{ 4 \sin^2(3\gamma) / [9 - \sqrt{(81 - 72 \sin^2(3\gamma))}] \}^{1/2}$$

Asymmetric Rotor Model

- The Asymmetric Rotor Model (ARM) investigates rigid triaxial shapes

- The energies of the first two 2^+ states are:

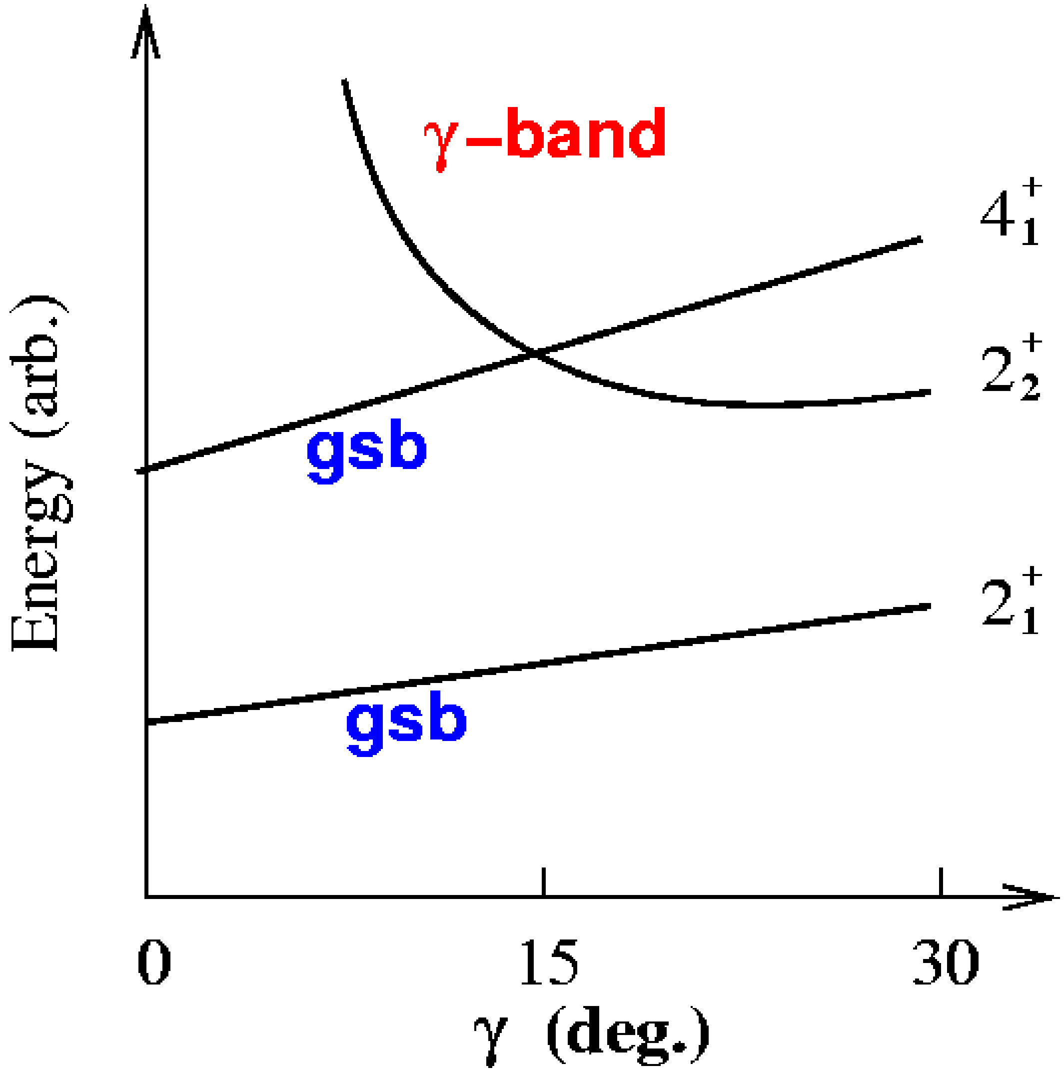
  $E(2^+) = (6\hbar^2/2\mathfrak{I})\,\{9[1 \pm \sqrt{(1 - 8/9\,\sin^2(3\gamma))}]\,/4\,\sin^2(3\gamma)\}$

- Hence from the experimental energies of the first 2^+ states, a value of $|\gamma|$ can be deduced

- The higher spin states of the ground-state and γ-bands move around in energy as γ changes

- <u>Increasing</u> γ tends to <u>lower</u> the energy levels of the γ-band relative to the ground-state band

ARM Energy Levels



- These are the lowest energy levels of an asymmetric rigid-rotor predicted by the ARM
- Note that the second 2^+ state falls below the first 4^+ state for

$|\gamma| \geq 15°$

More ARM Relations

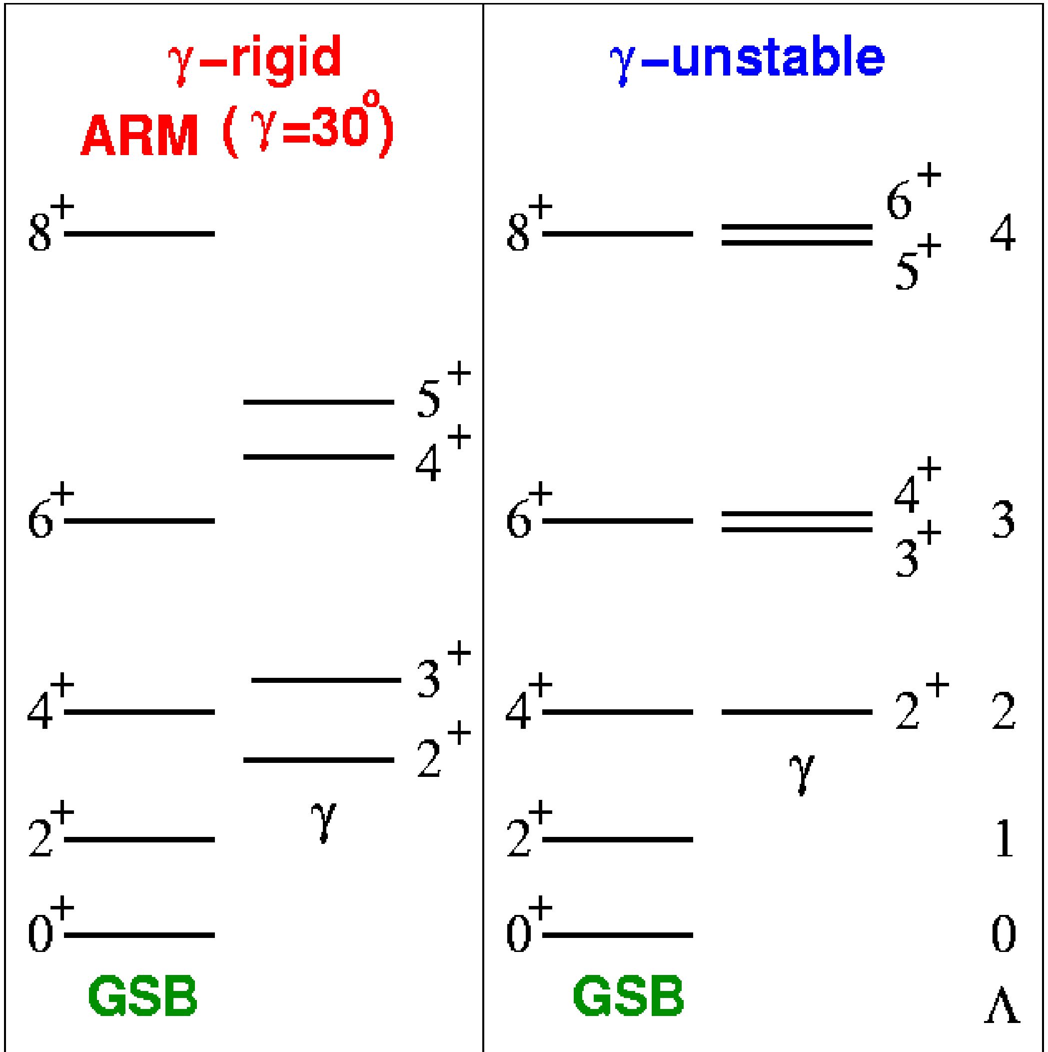
- For the odd-spin members of the γ band:

$E(3^+) = E_1(2^+) + E_2(2^+)$ and $E(5^+) = 4\ E_1(2^+) + E_2(2^+)$

- Percentage differences for N = 76 isotones:

Nucleus	$\lvert\gamma\rvert$	R_3 (%)	R_5 (%)
^{128}Te	26.6°	-1.21	
^{130}Xe	27.6°	+1.55	
^{132}Ba	26.3°	-0.98	
^{134}Ce	25.3°	-1.79	
^{136}Nd	25.7°	+0.38	+13.2
^{138}Sm	27.0°	+0.79	+18.7
^{140}Gd	26.8°	-2.53	+16.5

γ-rigidity or γ-softness?



- The ARM considers the rotation of a rigid triaxial shape
- The other extreme is a completely flat potential with respect to γ, with γ oscillating uniformly between $\gamma = 0°$ (prolate) and $\gamma = 60°$ (oblate)
- Since the average is $\gamma = 30°$, we compare the two models at this value

Gamma-Band Staggering

- As γ increases, a staggering arises between the odd-spin and even-spin members of the band
- One way to measure this is to form the ratio:

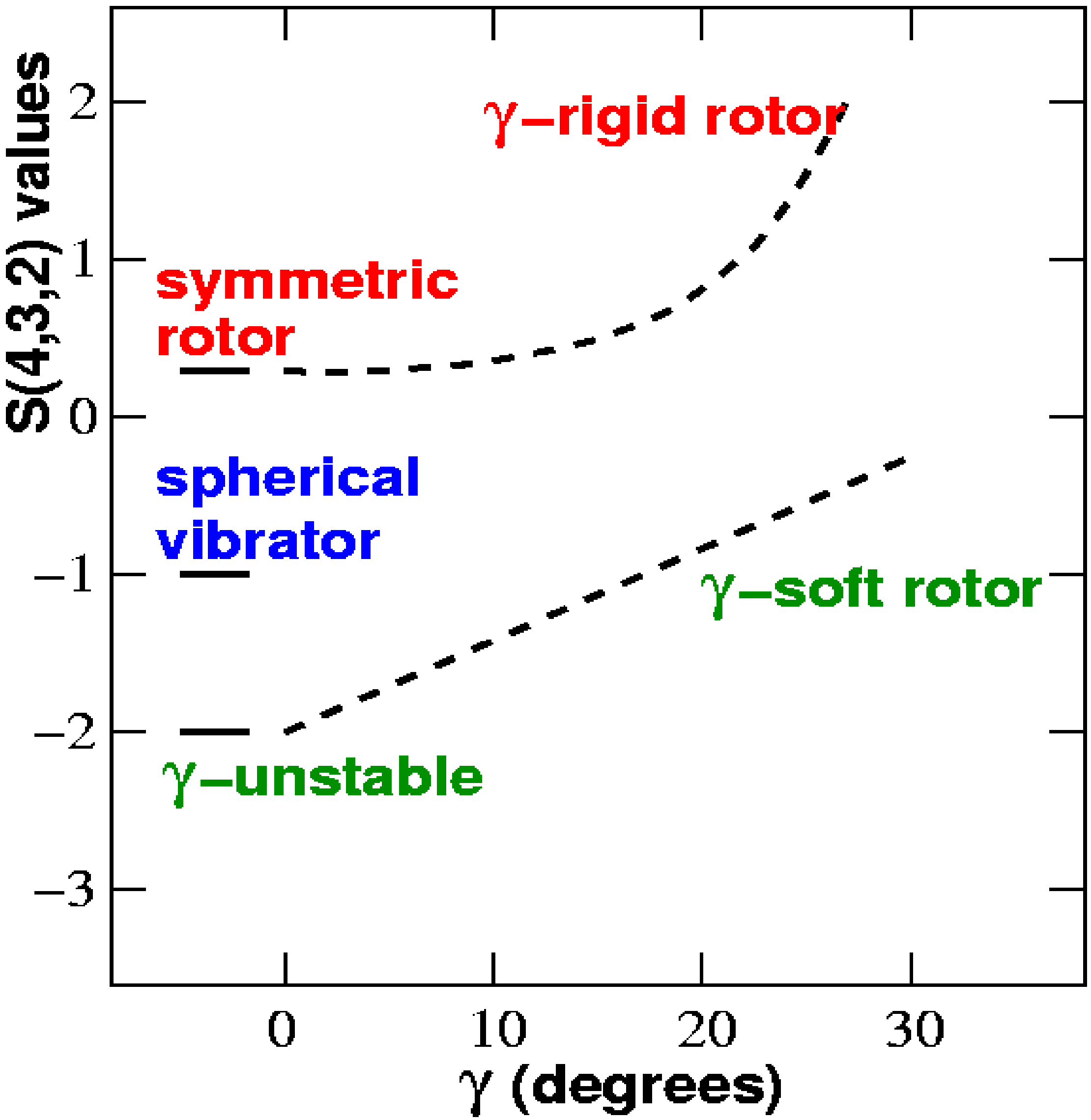
$$S(4,3,2) = \{ [E_2(4^+) - E_1(3^+)] - [E_1(3^+) - E_2(2^+)] \} / E_1(2^+)$$

- The energies, in units of $E_1(2^+)$, are:

Model	$E_2(2^+)$	$E_1(3^+)$	$E_2(4^+)$	S(4,3,2)
γ-rigid (30°)	2.0	3.0	5.67	+1.67
γ-unstable	2.5	4.5	4.5	-2.0

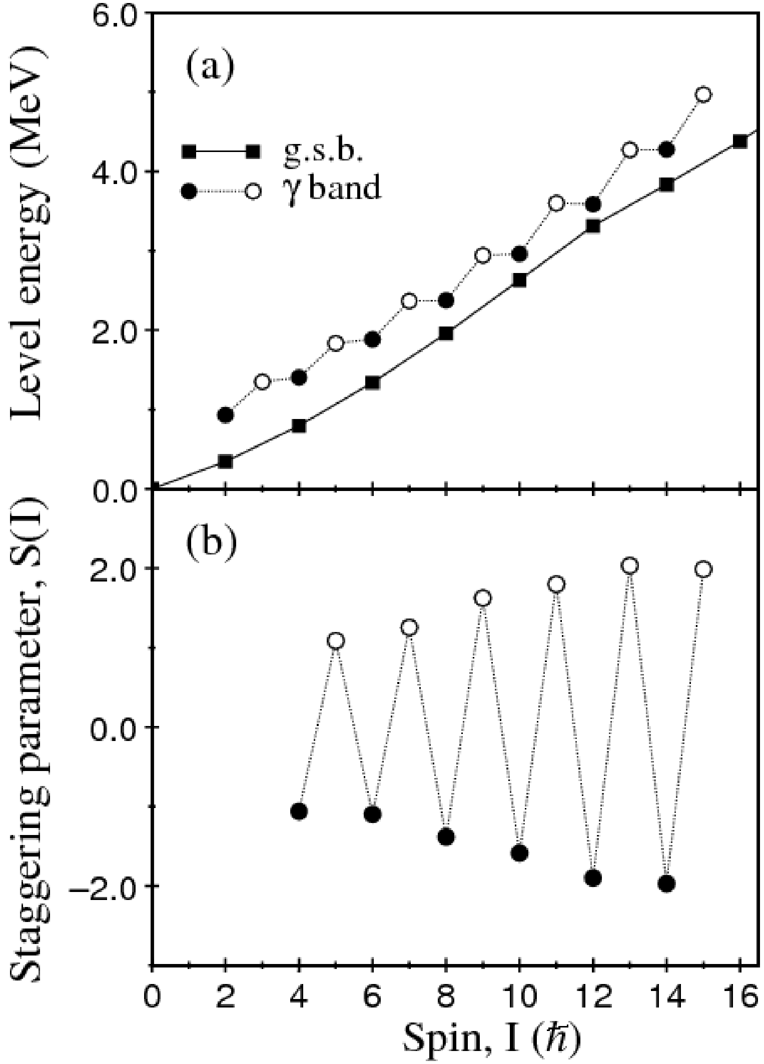
- Spherical Harmonic Vibrator: S(4,3,2) = -1.0
- Symmetric Rotor (γ = 0°): S(4,3,2) = +0.33

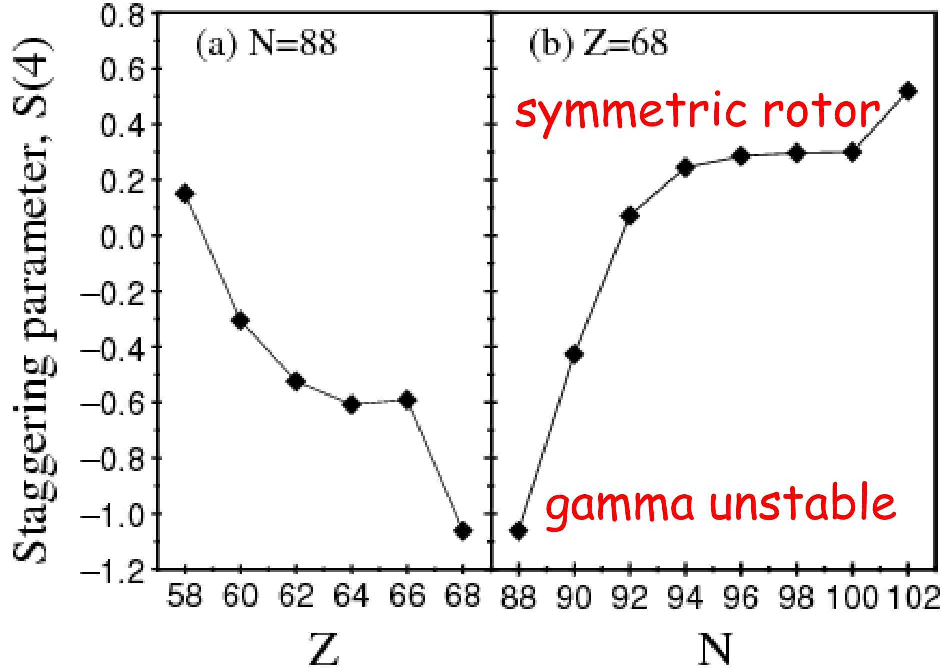
S(4,3,2) Ratios vs. γ



- The S(4,3,2) values for various types of motion are shown here as a function of the γ deformation

Staggering in ^{156}Er





^{156}Er (Z=68, N=88) is a 'transitional' nucleus

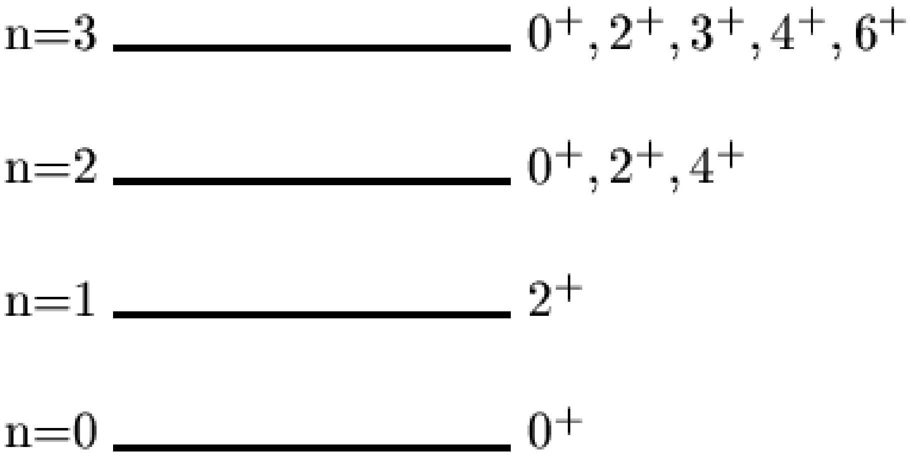
Do Triaxial Nuclei Really Exist?

- Considerable effort has been made over the last twenty years to obtain conclusive evidence of (static) triaxial nuclear shapes
- These efforts have recently been intensified by the experimental evidence of chirality (handedness) and the wobbling (precession) mode in nuclei (discussed later)
- Triaxiality is an essential prerequisite for the manifestation of both of these effects in the atomic nucleus !

Vibrational Motion

- Spherical Harmonic Vibrator

Spherical Harmonic Vibrator



- A dynamic deformation
- We assume the nucleus is spherical in its ground state and the excited states are due to harmonic oscillations of the nuclear surface

- For a quadrupole vibration, the potential may be written:

$$V_{vib} = \sum_{\mu} \{\tfrac{1}{2} C_2 |\alpha_{2\mu}|^2 + \tfrac{1}{2} B_2 \, |d\alpha_{2\mu}/dt|^2\}$$

where C_2 is a parameter representing the restoring potential and B_2 is associated with the mass carried by the vibration. This mode is possible since C_2, determined by the surface tension, is low

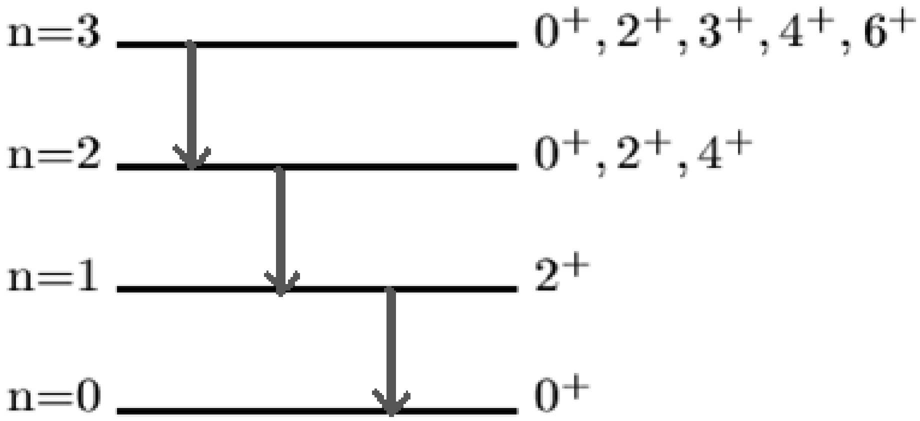
Vibrator Eigenvalues

- The eigenvalues of the spherical harmonic vibrator are:

$$E_n = E_0 + n\,\hbar\omega_2 \quad \text{with} \quad \omega_2 = \sqrt{(C_2/B_2)}$$

- E_0 represents the intrinsic and zero point motion of the oscillations
- The energy levels for different n are equally spaced
- Each phonon carries angular momentum 2 (Y_2) and has positive parity

Allowed Transitions



- For quadrupole vibrations, electromagnetic transitions are only allowed between states with:

$$\Delta n = \pm 1$$

- For an E2 transition:

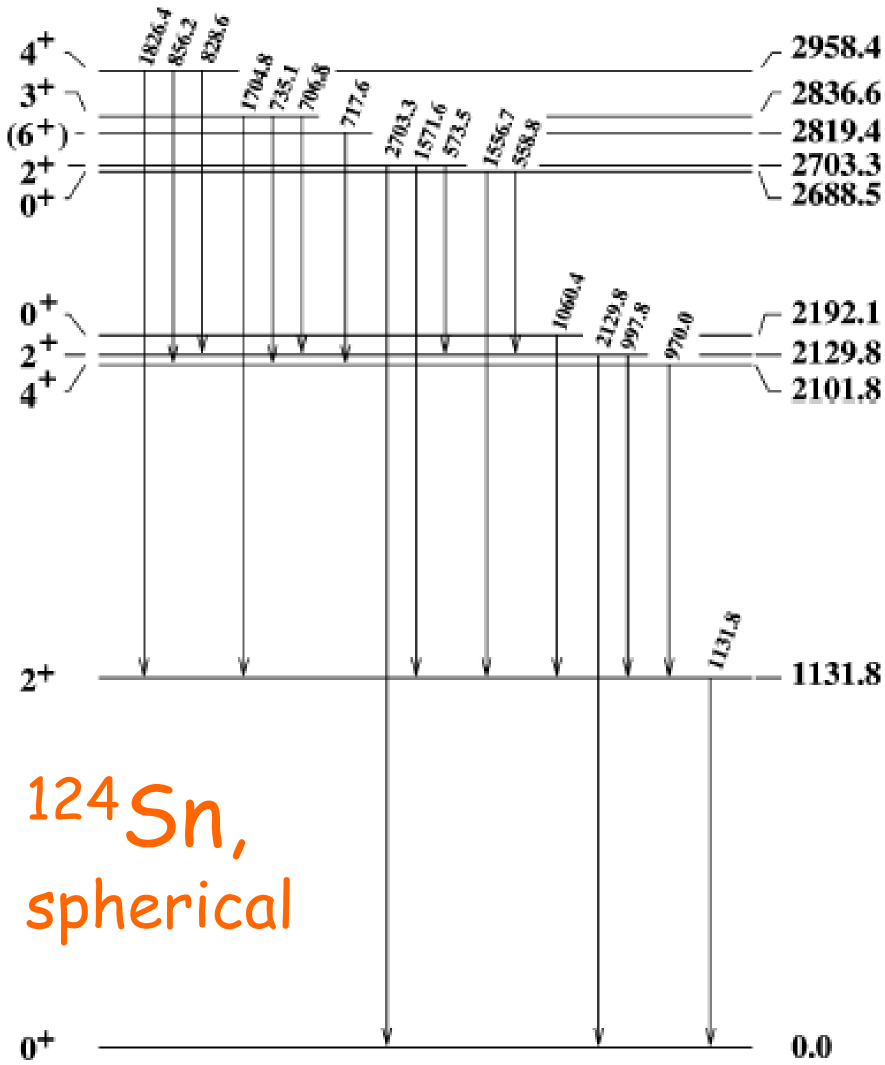
$$\langle I=2, n=1 || E2 || I=0, n=0 \rangle = \sqrt{(5)}\, Q_{vib} e$$

where Q_{vib} is calculated from the Liquid Drop Model:

$$Q_{vib} = (3ZR^2/4\pi) \sqrt{(\hbar/2B_2\omega_2)}$$

- The magnetic moment is constant for $\lambda = 2$ states and therefore M1 transitions are not allowed

Multiphonon Vibrational States



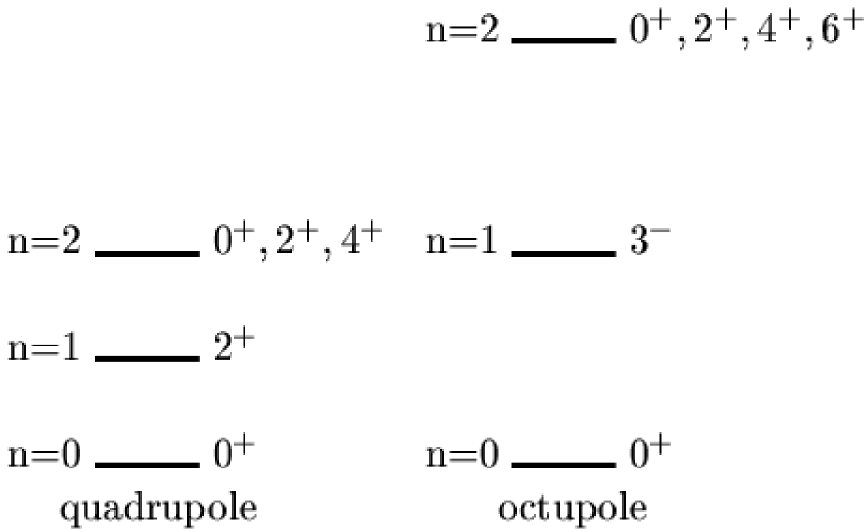
N = 3 (3 phonon)

N = 2 (2 phonon)

N = 1 (1 phonon)

N = 0

Octupole Vibrations



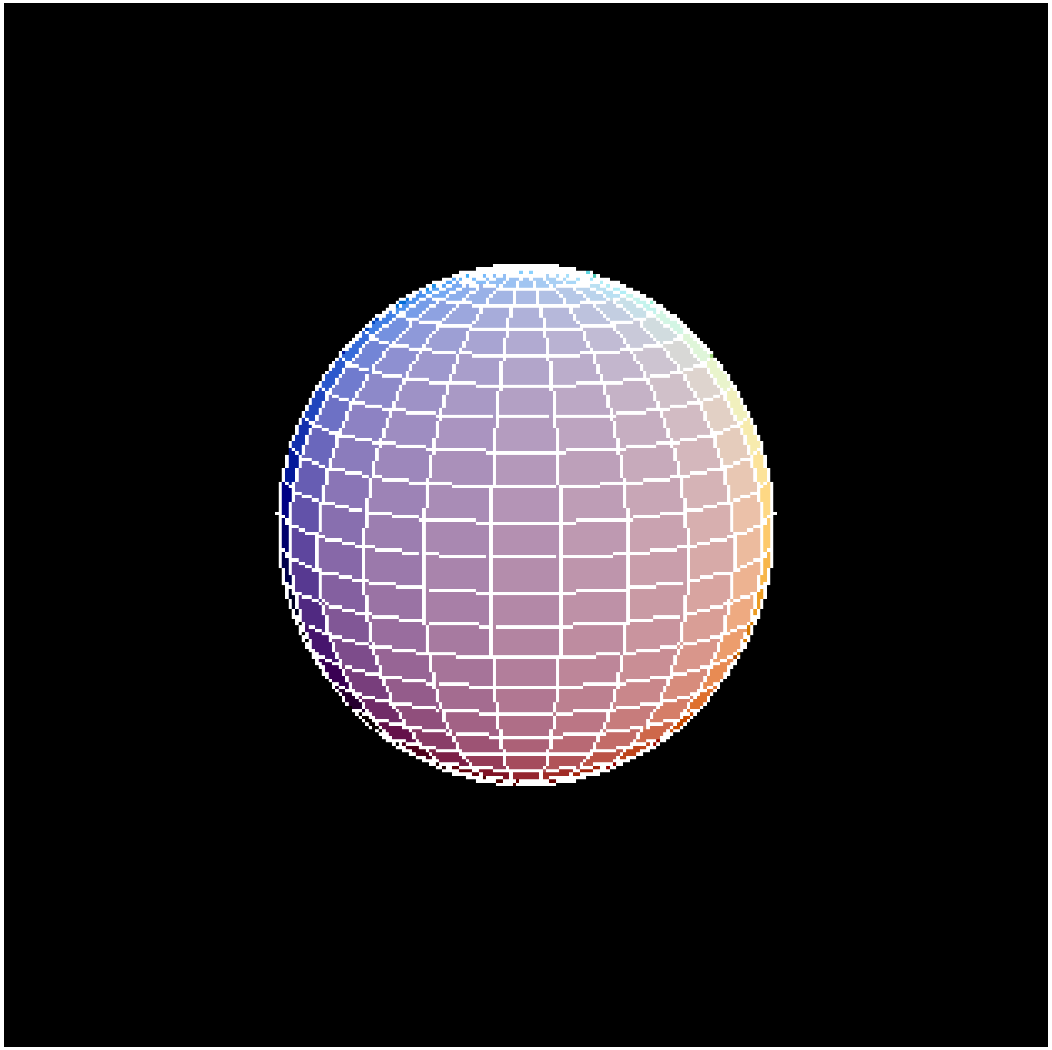
- For octupole vibrations, each phonon carries angular momentum 3 (Y_3) and negative parity

- The energy of the first excited state (3^-) is roughly twice the energy of the quadrupole case

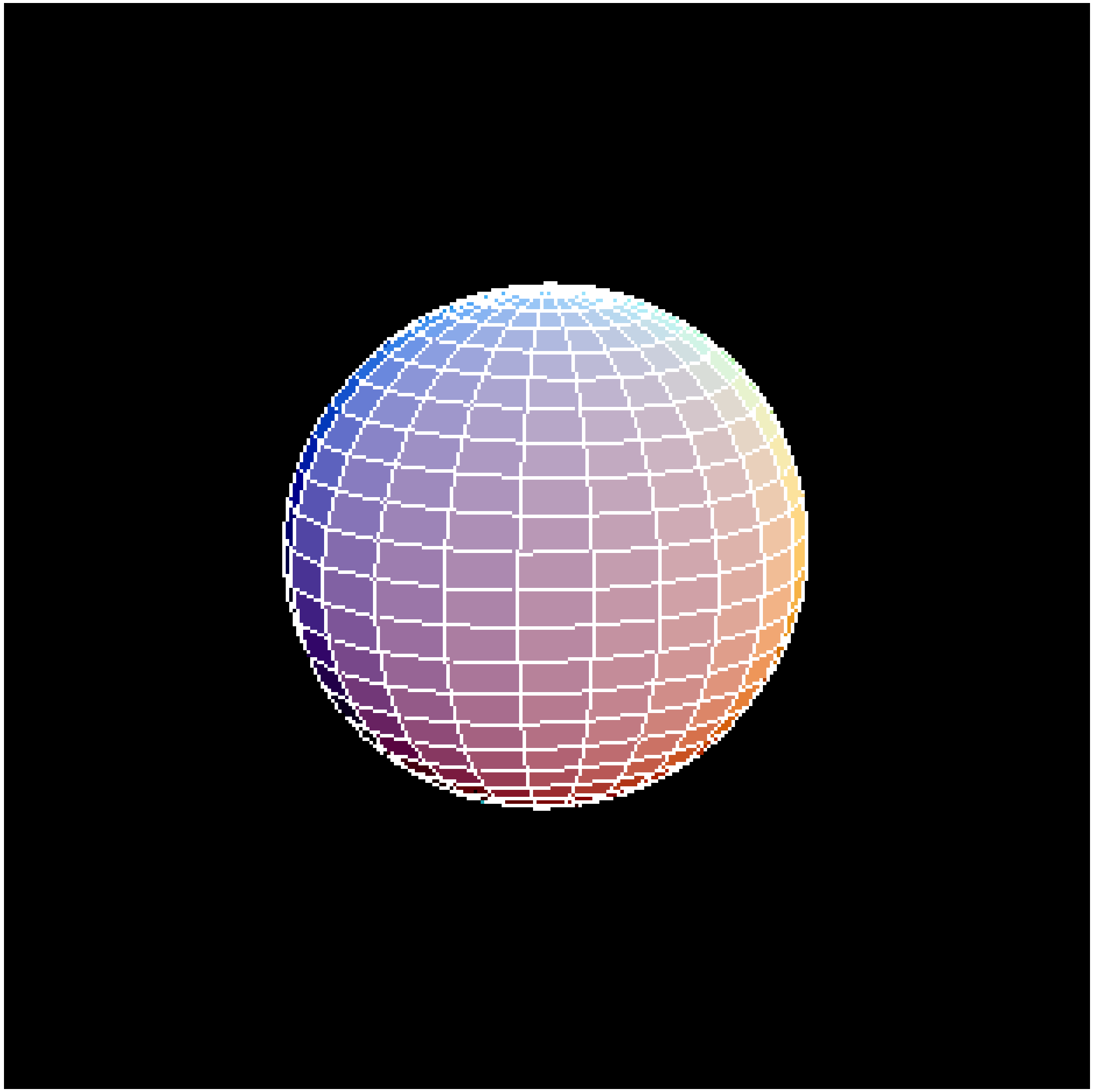
- For real nuclei, an anharmonic oscillator is needed. This removes the degeneracy of the n = 2 states (0^+, 2^+, 4^+) of the quadrupole vibrator. It also displaces the $\lambda = 2$ and $\lambda = 3$ states relative to each other

Vibrational Movies…

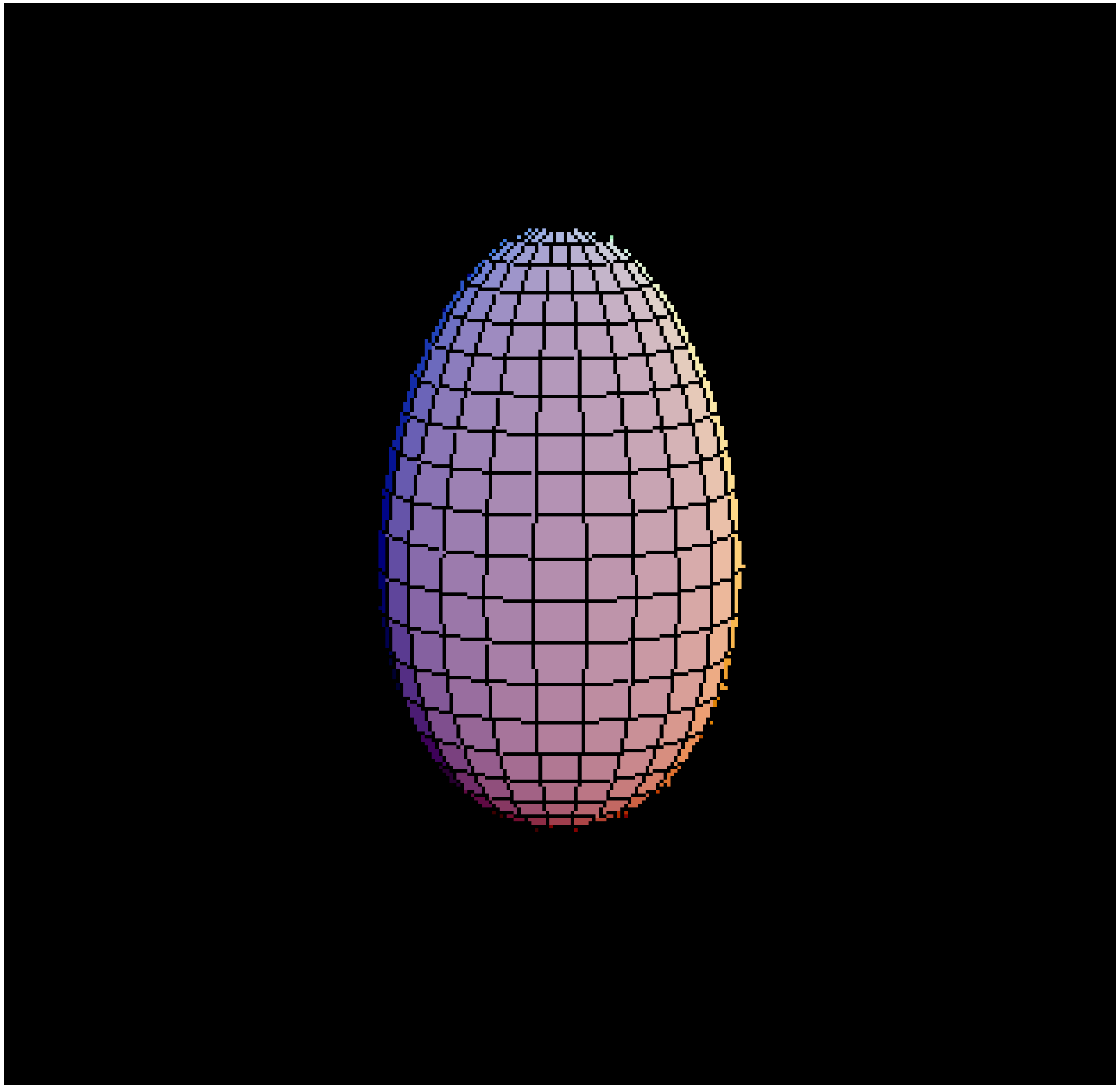
Beta (Y_{20}) Vibration



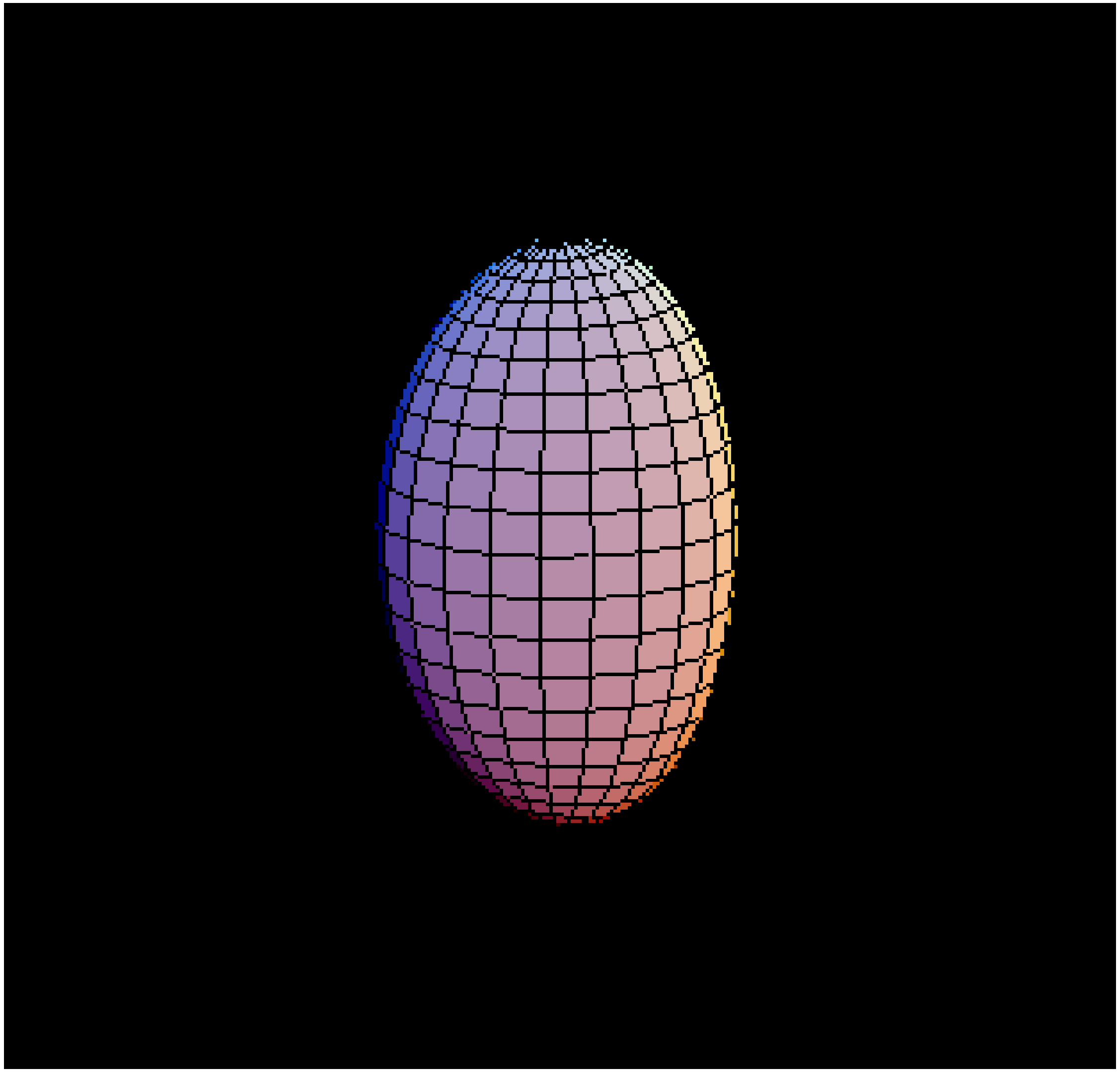
Gamma (Y_{22}) Vibration



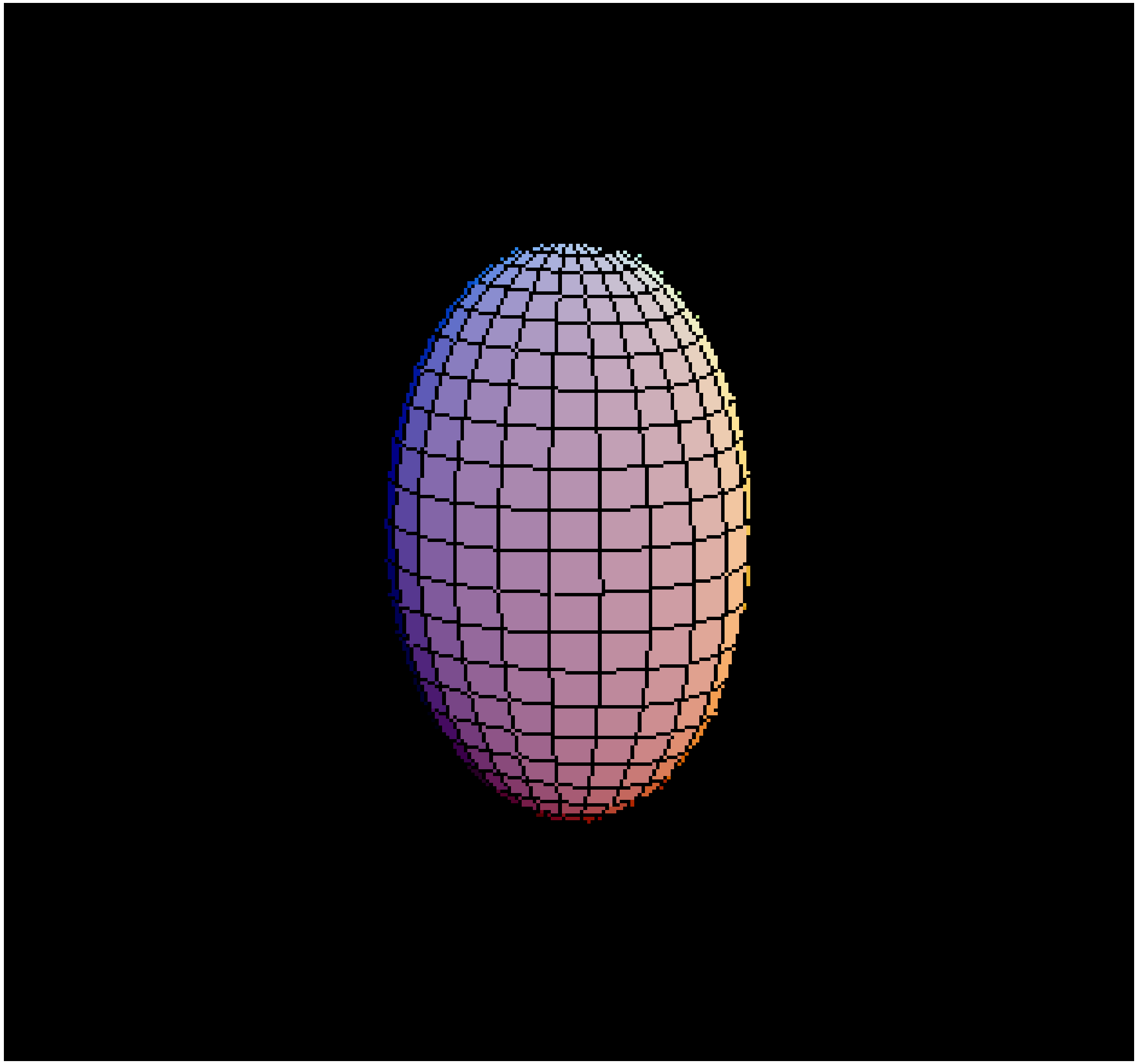
Octupole (Y_{30}) Vibration



Octupole (Y_{31}) Vibration



Octupole (Y_{32}) Vibration



Octupole (Y_{33}) Vibration